N_{ReG} = superficial gas Reynolds number
N_{ReL} = superficial liquid Reynolds number
R = pipe radius, in.
v_z = axial velocity of the film, ft./sec.
v_θ = circumferential velocity of the film, ft./sec.
z = coordinate direction parallel to the pipe axis positive direction is downstream, in.
δ = film thickness, in.
θ = angular direction, 0 deg. at the top of the pipe, 180 deg. at the bottom of the pipe, deg. or rad.
ρ_L = liquid density, lb./cu.ft.

Subscripts and Superscripts

exp = experimentally determined value of a variable
pre = value of a variable calculated from a proposed model
1 = coordinate position
2 = coordinate position where 2 is greater than 1
bar = average value of a variable with respect to a coordinate direction
tick = instantaneous value of a variable

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Continuous Filter Cake Washing Performance

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A previous report (1) has presented the derivation of new filter cake washing equations. This report presents the procedure for their application and contains an example of application to a specific batch washing system. It also presents the adaption of the equations to permit their use in predicting the performance of a continuous rotary-drum filter.

Filtration accomplishes the removal of undissolved solid material from a liquid by passing slurries through a filter medium in either a continuous or discontinuous operation.

In many slurries, the filtrate contains dissolved material, called the *solute*.

If the filtrate retained in the filter cake is not removed before further operations, such as drying, this dissolved material will remain in the cake as the liquid is removed. Washing is used for the recovery or reduction of solute content in the cake. In the study of continuous washing processes, the mechanism of cake formation as well as cake washing performance must be well known.

Choudhury and Dahlstrom (*2*) employed Rhodes washing equation (*4*) to describe washing performance in a continuous rotary-drum filter operation. From commonly accepted filtration theory, the wash time is correlated as a function of wash ratio with parameters of cake formation time. (Note: wash ratio R is defined as

$$R = \frac{\text{volume of wash liquor}}{\text{volume of original liquor in cake}}.\Big)$$

Their correlation of recovery ratio as a function of wash ratio appears to be satisfactory. (Note: recovery ratio = weight fraction of solute remaining in cake after washing.)

A new filter cake washing performance equation has been derived (*1*). It assumes plug flow of the washing liquor through pore channels in the filter cake. The filtrate to be recovered is in the cake as a stagnant film on the

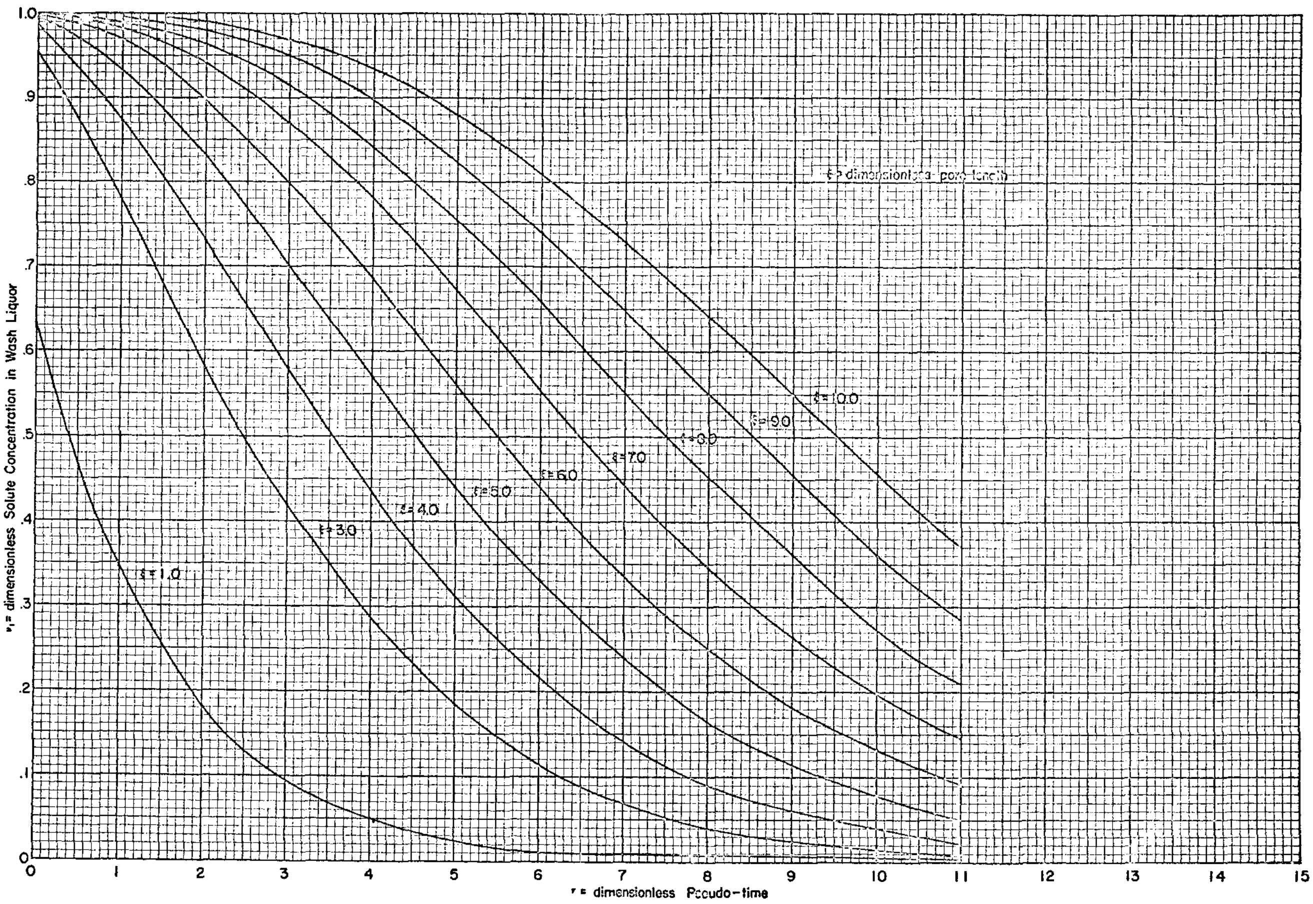

Fig. 1. Solute concentration in wash liquor.

channel surface. Its solute is removed by the wash liquor as a result of mass transfer from the stagnant channel surface film to the washing liquor.

This report shows the application of the equations to experimental data obtained on a calcium chloride solution. It also shows the adaptation of the equations to predict the performance of a continuous rotary-drum filtration system.

FILTER CAKE WASHING EQUATIONS

The two equations which have been developed (*1*) to characterize a filter cake washing system are

$$\left(\frac{C_1-\alpha}{\beta-\alpha}\right) = 1 - e^{-(\xi+\tau)}\, I_0(2\sqrt{\xi\tau}) - \int_0^\tau e^{-(\xi+\eta)}\, I_0(2\sqrt{\xi\eta})\, d\eta \quad (1)$$

$$\left(\frac{C_2-\alpha}{\beta-\alpha}\right) = 1 - \int_0^\tau e^{-(\xi+\eta)}\, I_0(2\sqrt{\xi\eta})\, d\eta \quad (2)$$

Equation (1) provides the dimensionless concentration of the solute in the wash liquor and Equation (2) the dimensionless concentration of solute in the filtrate film.

We assumed that before the start of a wash cycle, most of the filtrate has been forced out of the filter cake pore spaces by the pressure difference across the filter cloth. Channels for the flow of wash liquor are thus formed. The washing serves to extract the remaining solute from this film.

To adapt these equations to describe a particular system, it is necessary to estimate values for their parameters, h_1L/U, h_2/h_1, h_2, which give the closest agreement between concentrations predicted by the equations and experimentally observed concentrations. The experimental data required can be obtained from batch filter cake washings in the laboratory. The data should show the concentration of solute in the wash liquor leaving the cake as a function of washing time. Runs should be made at various levels of cake thickness (L) and wash liquor flow (Q). A good pattern of runs is a two level factorial in log Q and log L with a replicated center point to provide an estimate of error.

PROCEDURE FOR ESTIMATING THE PARAMETERS

Equations (1) and (2) are sufficiently complex that they are most readily used by referring to families of curves representing their solutions for various values of ξ, ν_1, ν_2, and τ. Figure 1 shows the generalized solutions for Equation (1) and Figure 2 for Equation (2). To use these figures, the experimental data are transformed into the dimensionless form of Figure 1 by using estimated transforming parameters. By repeated estimates of the transforming parameters, the transformed experimental data are adjusted until the form of their plots agree most nearly with the curves of the generalized solution. The desired value of the equation parameter can then be read from the curve.

The details of this procedure, which is used to analyze each experimental run, are outlined in the following steps:

1. Guess a value for the wet base void fraction ϵ. Values normally lie between 0.3 and 0.4.
2. Compute an estimate of L/U from

$$L/U = \frac{L\epsilon}{Q/A}\ \text{sec.}$$

3. Guess a value for h_2.
4. Calculate the transformed values of τ and ν_1 for the data C_1 and t by using the following equations:

$$\tau = h_2(t - L/U)$$

$$\nu_1 = \frac{C_1 - \alpha}{\beta - \alpha}$$

5. Plot ν_1 vs. τ and compare the form of the curve obtained with the curves in Figure 1. If the form of the curves match, read off the value for ξ. If the form of the curves do not match, guess value of h_2 and repeat steps 3 to 5. It may be necessary to adjust ϵ as well as h_2 in order to get satisfactory agreement. This procedure will provide values for L/U, h_2, and ξ.
6. Calculate h_1 from

$$h_1 = \frac{\xi}{L/U}$$

7. Perform steps 1 through 6 on all sets of experimental data. Make plots of h_1L vs. Q/A and h_2L vs. Q/A on log-log paper and fit a straight line through the points. These graphs provide a source of values for h_1 and h_2 and, with the value obtained for the constant ϵ, provide a way to calculate the performance of the filter cake washing system.

The estimates of the various parameters obtained by analyzing data from five experimental runs in this manner are given in Table 1.

Figures 3 and 4 show the relationship between h_1L, h_2L, and Q/A. Figures 5 through 9 show the curve predicted by Equation (1) vs. the observed data. These experimental data were obtained on a system of calcium carbonate cake containing an ammonium chloride solution and washed with water.

APPLICATION TO CONTINUOUS WASHING SYSTEMS

Equations (1) and (2) developed for the batch washing situation can be adapted to predict the performance of a continuous filter cake washing system of the general type shown in Figure 10. In this system, as the filter cake moves through the washing zone, and the solute is washed from it and flows out with the wash liquor at a concentration $\overline{C}_1$. In the filter cake, the initial solute concentration is β, and the final average concentration after washing is $\overline{C}$. The total washing time T is a function of the cake velocity through the washing zone and the washing zone length.

The adaption of Equations (1) and (2) to this continuous case results in

$$\overline{C}_1 = \frac{1}{T}\left[\beta\left(\frac{L}{U}+\frac{G}{h_2}\right)+\frac{\alpha}{h_2}(\overline{\tau}-G)\right] \text{when}\quad T > \frac{L}{U} \tag{3}$$

$$\overline{C}_1 = \beta \qquad \text{when } T < \frac{L}{U}$$

where

$$G = \int_0^{\overline{\tau}} \left(\frac{C_1-\alpha}{\beta-\alpha}\right) d\tau \tag{4}$$

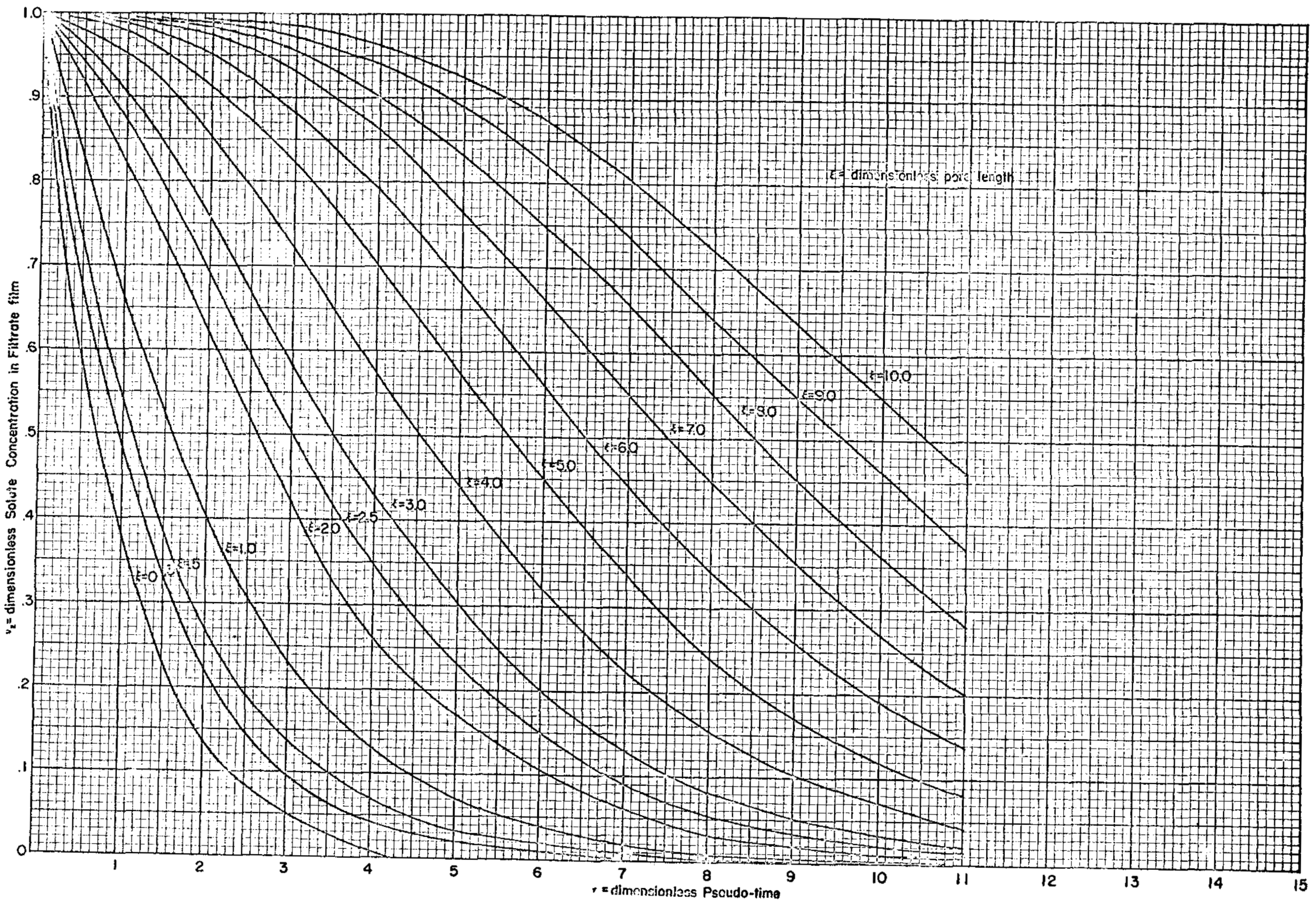

Fig. 2. Solute concentration in filtrate film.

Table 1.

Run number	ϵ	L/U (sec.)	ξ	h_1 (1/sec.)	h_2 (1/sec.)
1	0.35	110.6	6.0	0.0542	0.0475
2	0.40	141.1	6.0	0.0425	0.047
3	0.36	84.8	6.0	0.0708	0.08
4	0.34	47.5	6.0	0.1263	0.105
5	0.39	55.1	6.0	0.1089	0.155
Average	0.37	—	6.0	—	—

and

$$\overline{\tau} = h_2 \left(T - \frac{L}{U} \right) \tag{5}$$

The parameter ξ is evaluated by using the following equation:

$$\xi = \frac{h_1 L}{U} \tag{6}$$

A measure of a filter cake washing system performance can be defined as the percent recovery E which is the amount of solute recovered in the wash liquor expressed as a percent of the solute that ideally could be recovered by a wash liquor having an initial solute concentration of α.

To calculate the percent recovery E, it is necessary to determine the amount of solute originally in the cake. This can be done by using the relationship that the wet base void fraction ϵ is proportional to A_1, while the dry base void fraction ϵ' is proportional to $(A_1 + A_2)$:

$$\frac{\epsilon}{A_1} = \frac{\epsilon'}{A_1 + A_2}$$

and

$$hS = h_1 A_1 = h_2 A_2$$

Therefore

$$\epsilon' = \epsilon(1 + h_1/h_2) \tag{7}$$

By using Equation (7), the value for ϵ' in Table 2 was calculated from values of h_1 and h_2 from Figures 3 and 4.

The values with the asterisk were computed directly from the experimental data as a check on the validity of the procedure.

From Equation (7) for ϵ', the amount of liquor initially in the cake is

$$W_0 = \frac{AL\,\epsilon'}{A} \quad \text{cu.ft./sq.ft.} \tag{8}$$

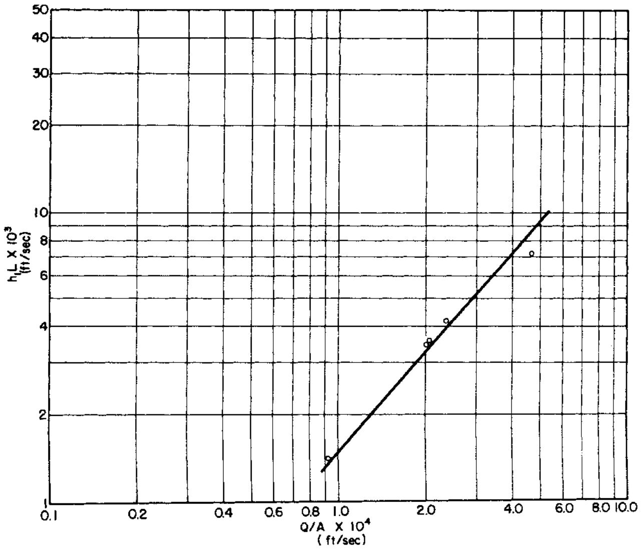

Fig. 3. Modified mass transfer coefficients vs. wash liquor flow rate.

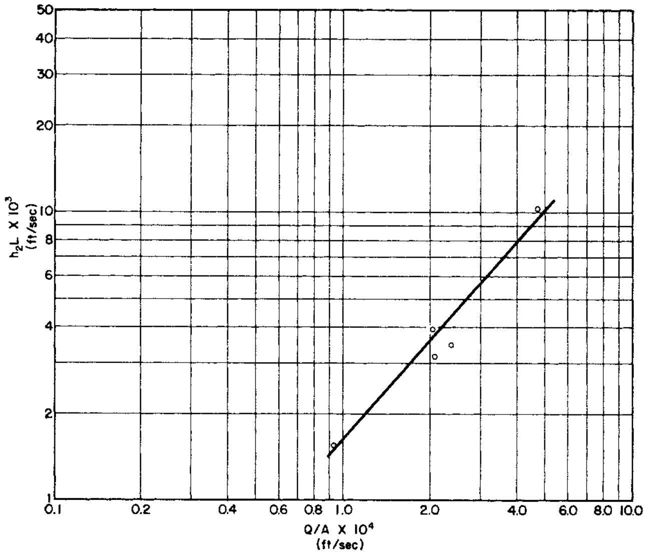

Fig. 4. Modified mass transfer coefficients vs. wash liquor flow rate.

From material balance considerations, the average concentration of solute remaining in the filter cake $\overline{C}$ is

$$WT(\alpha - \overline{C}_1) = W_0(\overline{C} - \beta)$$

Solving for $\overline{C}$, we get

$$\overline{C} = \frac{WT}{W_0}(\alpha - \overline{C}_1) + \beta \tag{9}$$

In Equation (9), W is the wash liquor flow rate and is equal to Q/A. The percent recovery can now be expressed as

$$E = \left(\frac{\beta - \overline{C}}{\beta - \alpha} \right)(100) \tag{10}$$

From Equation (9) this can be expressed in terms of $\overline{C}_1$ as

$$E = \frac{WT}{W_0}\left(\frac{\overline{C}_1 - \alpha}{\beta - \alpha} \right)(100) \tag{11}$$

NUMERICAL EXAMPLE

Consider a continuous washing system of the kind shown in Figure 10. From previous laboratory work, the following parameter values have been determined:

$\epsilon = 0.35$

h_1L (see Figure 3)

h_2L (see Figure 4)

Table 2.

Run No.	ϵ	ϵ'	$(\epsilon')^*$
1	0.35	0.749	0.732
2	0.40	0.762	0.736
3	0.36	0.680	0.685
4	0.34	0.749	0.713
5	0.39	0.664	0.632

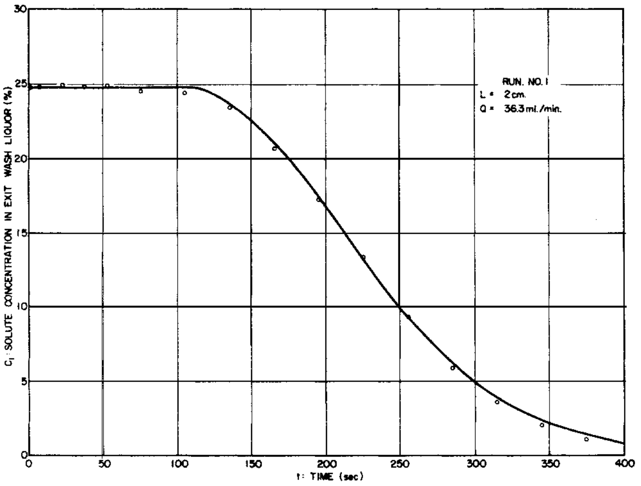

Fig. 5. Solute concentration in exit wash liquor.

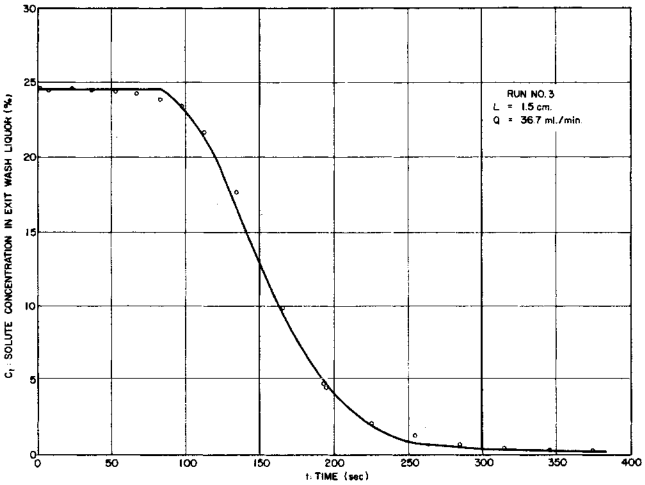

Fig. 7. Solute concentration in exit wash liquor.

The concentrations of solute in the washing liquor and in the cake for this system are

$$\alpha = 0$$

$$\beta = 0.23 \text{ mole/cu.ft.}$$

From the equations of the previous section, the relationship between percent recovery and wash ratio can be determined for any given washing time.

Assuming the following values, we get

$$T = 2 \text{ min.} = 120 \text{ sec.}$$

$$Q/A = 2.5 \times 10^{-4} \text{ cu.ft./sq.ft./sec.}$$

$$L = 0.60 \text{ in.} = 0.05 \text{ ft.}$$

Therefore

$$L/U = \frac{L\epsilon}{Q/A} = \frac{(0.05)(0.35)}{(2.5 \times 10^{-4})} = 70 \text{ ft./ft./sec.}$$

From Figures 3 and 4

$$h_1L = 4.1 \times 10^{-3} \text{ ft./sec.}$$

$$h_2L = 4.5 \times 10^{-3} \text{ ft./sec.}$$

Therefore

$$h_1 = 0.082 \text{ 1/sec.}$$

$$h_2 = 0.090 \text{ 1/sec.}$$

and

$$\xi = \frac{h_1L}{U} = (0.082)(70) = 5.74$$

$$\bar{\tau} = h_2\left(\mathrm{T} - \frac{L}{U}\right) = (0.09)(120 - 70) = 4.5$$

$$G = 3.74$$

From Equation (3)

$$\bar{C}_1 = \frac{1}{T}\left[\beta\left(\frac{L}{U} + \frac{G}{h_2}\right) + \frac{\alpha}{h_2}(\bar{\tau} - G)\right]$$

$$= \frac{1}{120}\left[0.23\left(70 + \frac{3.74}{0.09}\right)\right] = 0.222 \text{ mole/cu.ft.}$$

This is the concentration of the leaving wash liquor stream.

For the given Q/A and L, the wash ratio is

$$R = \frac{WT}{W_0} = \frac{Q}{A}\left(\frac{T}{L\epsilon(1 + h_1/h_2)}\right)$$

$$= \frac{(2.5 \times 10^{-4})(120)}{(0.05)(0.35)\left(1 + \frac{0.082}{0.090}\right)} = 0.895$$

The percent recovery of solute predicted from Equation (11) is

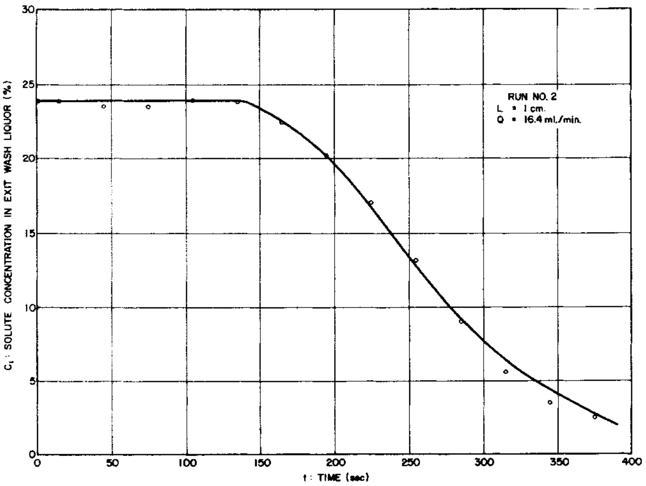

Fig. 6. Solute concentration in exit wash liquor.

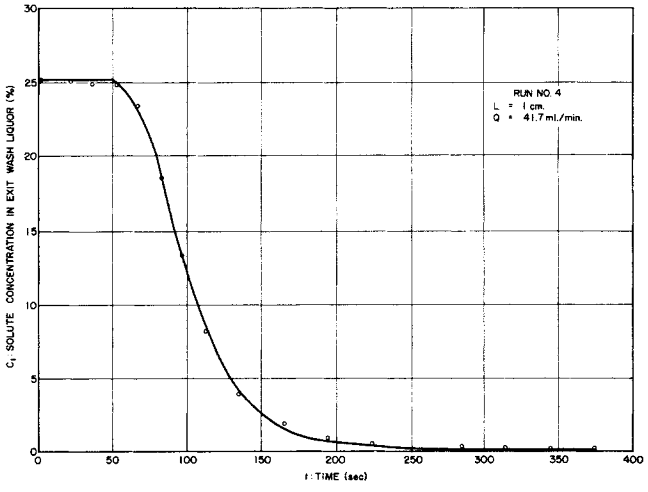

Fig. 8. Solute concentration in exit wash liquor.

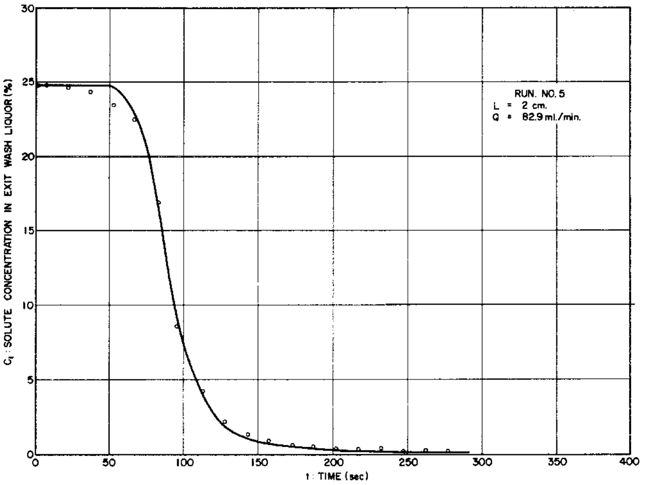

Fig. 9. Solute concentration in exit wash liquor.

$$E = \frac{WT}{W_0}\left(\frac{\overline{C}_1 - \alpha}{\beta - \alpha}\right)(100)$$

$$= (0.895)\left(\frac{0.222}{0.23}\right) = 86.5\%$$

COMMENT

This report demonstrates an application of a washing model (*1*) to analyze a continuous filter cake washing operation.

In this cake washing model (*1*), a plug flow was assumed in the idealized pore channels within the cake. Thus, the resultant equations would be a representation of an ideal rather than a practical one. However, the equations would serve as a guide to an analysis of a network of continuous filtration units and to obtain a better operating condition in the practical filtration operation.

As Dahlstrom (*3*) has pointed out, in the practical continuous filter cake washing one would always experience some retention of solute in the cake after every washing. In other words, the percent recovery E is always less than 100%. Two factors may be considered: the concentration of solute in the stagnant film reaches an equilibrium, and most likely, a fraction of solute is retained in the pockets between cake particles and would not be fully exposed to wash liquor. These factors should be further investigated through experimentation and theoretical development.

NOTATION

A = total area of filter cake perpendicular to the direction of wash liquor flow, sq.ft.
A_1 = cross-sectional area of a channel perpendicular to the direction of flow, sq.ft.
A_2 = cross-sectional area of film in a channel perpendicular to the direction of wash liquor flow, sq.ft.
$\overline{C}$ = average concentration of solute remaining in cake, mole/cu.ft.
C_1 = concentration of solute in wash liquor, mole/cu.ft.
C_2 = concentration of solute in stagnant film, mole/cu.ft.
$\overline{C}_1$ = average concentration of solute in wash liquor outlet, mole/cu.ft.
$G = \int_0^{\overline{\tau}} \left(\frac{C_1 - \alpha}{\beta - \alpha}\right) d\tau$

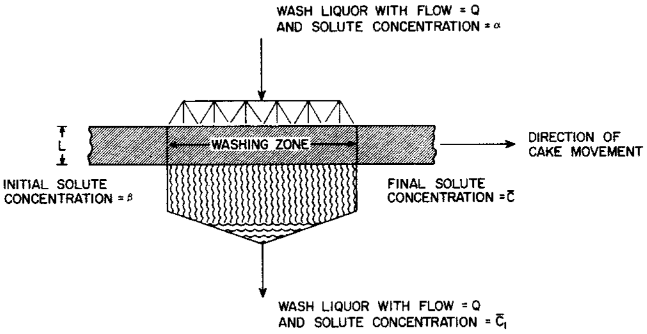

Fig. 10. Continuous cake washing system.

h = mass transfer coefficient, mole/(sq.ft.)(sec.)/mole/cu.ft.
$h_1 = \frac{hS}{A_1}$, $h_2 = \frac{hS}{A_2}$ = modified mass transfer coefficient, 1/sec.
I_0 = zero-order Bessel function with imaginary argument: $I_0(z) = J_0(iz)$
L = cake thickness, ft.
L/U = wash liquor residence time
Q = total volumetric flow rate of wash liquor, cu.ft./sec.
R = wash ratio, the volume of wash liquor divided by the volume of liquor initially in the cake
S = surface area in a channel for mass transfer, sq.ft./ft.
t = time from beginning of wash cycle, sec.
T = total washing time, sec.
U = linear velocity of wash liquor, ft./sec.
W = wash liquor volumetric flow rate, cu.ft./sq.ft./sec.
W_0 = volume of initial liquor in cake, cu.ft./sq.ft.

Greek Letters

α = inlet concentration of solute in the wash liquor, mole/cu.ft.
β = initial concentration of solute in the stagnant film, mole/cu.ft.
ϵ = wet base void fraction of the filter cake
ϵ' = dry base void fraction of the filter cake
$\nu_1 = \frac{C_1 - \alpha}{\beta - \alpha}$ = dimensionless concentration of solute in wash liquor
$\nu_2 = \frac{C_2 - \alpha}{\beta - \alpha}$ = dimensionless concentration of solute in filtrate film
$\xi = \frac{h_1 L}{U}$ = dimensionless distance from pore inlet
$\tau = h_2\left(t - \frac{L}{U}\right)$ = dimensionless time from arrival of wash liquor at pore distance L
$\overline{\tau}$ = overall dimensionless time

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